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Theoretical studies have been done on crack tip stress and deformation fields based on continuum plasticity for single crystals. Asymptotic analysis has been carried out for ideally plastic single crystals for both stationary and moving cracks under anti-plane shear and mode I plane strain conditions. They show that in the near tip region the plastic flow is confined primarily to planes along which both the stresses and the displacements are discontinuous for stationary cracks, or the velocity is discontinuous for quasi-statically growing cracks. These planes may be parallel or perpendicular to the traces of the slip planes which are locally stressed to yield levels and provide the plastic straining. The inclusion of strain hardening, as studied so far based on power hardening with resultant HRR fields, removes these discontinuities in the case of the stationary crack, although small angular ranges of very rapid stress variation result instead when there is only moderate strain hardening. In the ideally plastic limit however, these HRR fields yield a continuous displacement field, while resulting in the correct stress discontinuities. Thus, it is conjectured that the domain of validity of the HRR field must shrink as the ideally plastic limit is approached. Furthermore, this domain is expected to be confined to only the part of the plastic zone immediately adjacent to the crack tip. Studies are now underway to check these conjectures. Keywords: Crack Mechanics, Abstracts, Mechanical Properties, Shear Strain, Elasticity, Fracture (Mechanics), Weight Functions,

Full elastic-plastic solutions have been developed, using the finite-element method, for stationary and quasi-statically growing cracks in ideally plastic crystals under mode I plane strain conditions. These have been done for the small-scale-yielding problem and, also, for bend specimen geometries intended to simulate those used in our experiments. The elastic-ideally plastic finite-element solutions have nicely verified the discontinuous field predicted by the asymptotic analysis. They also suggest that these discontinuities are not restricted to the near tip region but exist, within a more diffuse plastic region, at distances of order of half the plastic zone size away from the tip.

The finite-element code for strain hardening material has also been formulated. The first runs, under preparation now, are designed to initially examine the region of validity of HRR fields as well as the effects of hardening on the flow pattern in the plastic region.

Furthermore, an asymptotic analysis has been carried out of the stress state near the tip of a rapidly moving crack in ductile crystals showing a strong viscoplastic effect, such that the Hart-Freund-Hutchinson conditions are met, under which an inverse square root type stress singularity is retained at the crack tip. We are attempting to evaluate the variation of energy release rate at the crack tip in the high strain rate regime. The goal of this work is to examine how high strain rates effect ductile to brittle responses of single crystals and of interfaces between crystals. At this stage, we are still in the process of accumulating experimental data to analyze our results.

In the experimental work precisely oriented single crystals of copper and iron-3% silicon have been grown and mechanically tested in the four-point bending configuration.

Dislocations, Tensile cracks, Crack Tips

(G)

Sharp cracks have been introduced in the specimens by first spark cutting and then fatigue cracking. Optical miore interferometry techniques have been used to measure the in-plane deformations at the specimen surfaces. The results of this work support the predicted theoretical fields but, together with the observed slip disruptions on well-polished specimen surfaces, suggest that the strongest features seen on the surfaces are governed by the local plane stress conditions. They involve zones of intense shearing that displace material points, relatively, in a direction perpendicular to the surface, and cannot result from plane flow. The surface distruptions do not necessarily predict the behavior of the interior of the specimen, but it is expected that the in-plane displacements at the surface will be closer to the behavior in the interior.

In addition fracture processes have been studied for copper bicrystals. The emphasis there has been on the transition from ductile response to brittle interfacial cleavage and the effect of grain boundary segregation, mainly of bismuth, and of sulphur too, in such crystals. The different fracture behaviors of these bicrystals has, to some extent, been explained in terms of their different grain boundary structures and the orientations of potentially relaxing slip planes. Such explanation has been based on comparison of the theoretically predicted value of the crack tip energy release rate, G, for dislocation emission from the crack tip against that for cleavage decohesion of the grain boundary. This approach, while very useful, does not account for the strong influence on crack tip response of the character of the surrounding plastic flow due to pre-existing dislocations, or dislocation sources, near the crack tip. Also, the experimental evidence suggests that the fracture surface may not precisely coincide, microscopically, with the grain boundary.

The copper bicrystals grown and examined in fracture tests, listed in order from highest to lowest ductility, are: a sigma 11 symmetrically tilted about the [110] direction with (1 -1 3) boundary plane, a sigma 9 symmetrically tilted about the [110] direction with (2 -2 1) boundary plane, a randomly oriented high angle boundary, and a sigma 5 symmetrically tilted about the [100] direction with (0 3 1) boundary plane. Embrittlement was induced by alloying with bismuth, which segregates to the grain boundary. The sigma 11 case is more resistent to impurity segregation than the sigma 9, and the random and the sigma 5 appear to be highly susceptible to segregation embrittlement. The ductility of the interfacial crack depends on the cracking direction. For example, the sigma 9 bicrystal appears to be fully ductile when the cracking direction is [-1 1 4], but fails in a brittle manner when the cracking direction is reversed to [1 -1 -4]. (Such striking differences, depending on the attempted cracking direction, were predicted from our modeling of dislocation nucleation.) These directional studies were done for what was thought to be pure copper, although sulphur segregation was found on the boundary plane.

Much work was also done on developing and applying three dimensional weight function theory in elastic crack analysis. This has enabled studies of crack front shapes that are perturbed from simple reference geometries (straight line or circle), of nonuniform crack growth through a regions of locally variable fracture resistance, and of crack tip ineractions with local heterogeneities, such as nearby transformed zones and dislocation loops emerging from a crack tip along a slip plane. The improved modeling of dislocation

loop nucleation from a crack tip, on the basis of weight function theory, has enabled us to resolve some long-standing uncertainties (on how to treat dislocation self energy) in that situation, and the results have been incorporated into studies by our group and others on the problem of ductile versus brittle response.

The weight function work in three-dimensional elasticity has led to results for the stress field and interaction energy for shear dislocation loops of arbitrary Burgers vector on a plane containing the crack tip, and oriented at an arbitrary angle with the crack plane. Those results have found immediate application elsewhere in our own work, in evaluating Rice-Thomson formulations of brittle versus ductile response. They have also been used by D. Clarke of IBM in interpreting his experiments on nucleation of dislocation loops from crack tips in silicon.

In addition, new elastic Green's function solutions have been derived for point loads near circular connections between half spaces. Results for cracks with fronts perturbed from the circular shape have now been derived for general shear and/or tensile loading of planar internal cracks and of cracks exterior to a connection between half spaces. The weight function work had partial support from USGS for its potential application to shear faults and, except for that type of application, further work on the topic (e. g., crack front trapping by tough obstacles) has been shifted to the UCSB/URI.

The subsequent section Publications summarizes by title the various manuscripts which have reported progress in the studies outlined above.

Publications (supported by ONR contract N00014-85-K-0405, from inception of project, 6/1/85, to end of contract, 11/30/89)

P. M. Anderson and J. R. Rice, "The Stress Field and Energy of a Three-Dimensional Dislocation Loop at a Crack Tip"; Journal of the Mechanics and Physics of Solids, **35**, 1987, pp. 743-769. ONR, NSF/MRL supported.

H. Gao, "Mismatched Elastic Connections", International Journal of Fracture, in press, 1990. ONR supported.

H. Gao, "Nearly circular shear mode cracks"; International Journal of Solids and Structures, **24**, 1988, pp. 177-193. ONR, USGS supported.

H. Gao, "Linear perturbation analysis of a shear loaded asperity", Journal of Geophysical Research, 94, 1989, pp. 10259-10266. ONR, USGS supported.

H. Gao, "Weight Functions for External Circular Cracks"; International Journal of

Solids and Structures, **25**, 1989, pp. 107-127. ONR supported.

H. Gao, "Application of 3D Weight Function - I. Formulation of Crack Interaction with Transformation Strain and Dislocations"; Journal of the Mechanics and Physics of Solids, **37**, 1989, pp. 133-153 ONR, UCSB/URI supported.

H. Gao and J. R. Rice, "Application of 3D weight Functions - II. The Stress Field and Energy of a Shear Dislocation Loop at a Crack Tip"; Journal of the Mechanics and Physics of Solids, **37**, 1989, pp. 155-174 ONR, UCSB/URI supported.

H. Gao and J. R. Rice, "Nearly Circular Connections of Elastic Half Spaces"; Journal of Applied Mechanics, **54**, 1987, pp. 627-634. ONR supported.

H. Gao and J. R. Rice, "Shear Stress Intensity Factors for a Planar Crack with Slightly Curved Front"; Journal of Applied Mechanics, **53**, 1986, pp. 774-778. ONR supported.

H. Gao and J. R. Rice, "Somewhat Circular Tensile Cracks"; International Journal of Fracture, **33**, 1987, 155-174. ONR supported.

R. Hill and J. R. Rice, "Discussion of 'A Rate-Independent Constitutive Theory for Finite Inelastic Deformation' by M. M. Carroll"; Journal of Applied Mechanics, **54**, 1987, pp. 745-747. ONR supported.

R. Nikolic and J. R. Rice, "Dynamic Growth of Anti-Plane Shear Cracks in Ideally Plastic Crystals"; Mechanics of Materials, **7**, 1988, pp. 163-173. ONR supported.

J. R. Rice, "Tensile Crack Tip Fields in Elastic-Ideally Plastic Crystals"; Mechanics of Materials, **6**, 1987, pp. 317-335. ONR, NSF/MRL supported.

J. R. Rice, "Two General Integrals of Singular Crack Tip Deformation Fields"; Journal of Elasticity, **20**, 1988, pp. 131-142. ONR supported.

J. R. Rice, "Weight Function Theory for Three-Dimensional Elastic Crack Analysis"; in Fracture Mechanics: Perspectives and Directions (Twentieth Symposium), ASTM, Philadelphia, 1989, pp. 29-57 ONR, UCSB/URI supported.

J. R. Rice, D. E. Hawk and R. J. Asaro, "Crack Tip Fields in Ductile Crystals"; International Journal of Fracture, 42, 1990, pp. 301-321 ONR supported.

J. R. Rice and M. Saeedvafa , "Crack Tip Singular Fields in Ductile Crystals with

Taylor Power-Law Hardening, I: Anti-Plane Shear"; Journal of the Mechanics and Physics of Solids, **36**, 1988, pp. 189-214. ONR supported.

M. Saeedvafa and J. R. Rice , "Crack Tip Singular Fields in Ductile Crystals with Taylor Power-Law Hardening, II: Plane Strain"; Journal of the Mechanics and Physics of Solids, 37, pp. 673-691, 1989 ONR supported.

J. - S. Wang and P. M. Anderson, "Fracture Behavior Embrittled fcc Metal Bicrystals and Its Misorientation Dependence"; Acta Metallurgica, in press ONR, NSF/MRL supported.

Ph. D. theses completed:

Peter M. Anderson, "Ductile and Brittle Crack Tip Response", September 1986. ONR, NSF/MRL supported.

Huajian Gao, "Applications of 3-D Weight Function Theory in Elastic Crack Analysis", April 1988. ONR, UCSB/URI, USGS supported.

Ruzica Nikolic, "Experimental Study of Crack Tip Processes and Plastic Flow in Ductile Crystals", January 1989. ONR, NSF/MRL supported

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Dr. Ruzica R. Nikolic, Graduate Student Research Assistant (Currently Faculty Member, Department of Mechanical Engineering, Technical University of Kragujevac, Yugoslavia)

Ms. Maryam Saeedvafa, Graduate Student Research Assistant

First pages (including abstracts) of published work:

First pages of publications, including title lines, abstracts in most cases, and portions of the introductory section have been copied onto the following pages.

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THE STRESS FIELD AND ENERGY OF A THREE-DIMENSIONAL DISLOCATION LOOP AT A CRACK TIP

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(*Received* 16 *February* 1987)

ABSTRACT

THE SELF STRESS field and self energy are estimated for a planar 3D dislocation loop emanating from a half-plane crack tip. While the problem is of greatest interest for analysis of shear loops nucleating from the crack tip in the concentrated stress field there due to applied loadings, it is addressed here in the interest of tractability for 3D prismatic loops lying in the same plane as the crack. Exact elastic calculations for that case are based on recent developments of 3D crack weight function theory and specific results are given for induced stress fields, intensity factors and energy of semicircular and rectangular prismatic dislocation loops. Also, self stresses and energy expressions are derived for the 2D case of a line dislocation lying parallel to the crack for arbitrary Burgers vector type and general orientation of the dislocated plane relative to the crack plane, and those results are used together with the 3D prismatic loop results to estimate approximately the self energy for 3D shear dislocation loops emanating from the tip on planes inclined to the crack plane. Energy results are given in terms of a correction factor m to the usual estimate of energy for an emergent crack tip loop as half the energy of a full loop (identified as the emergent loop and its image relative to the crack tip) in an uncracked solid. That is, if the energy of a full circular loop of radius r in an uncracked solid is $2\pi r A_0 \ln(8r/e^2 r_0)$, with r_0 = core cut-off and A_0 = energy factor, then the energy of a semicircular loop of radius r emerging from the crack tip is shown to take the form $\pi r A_0 \ln(8mr/e^2 r_0)$ and the constant m is calculated here as 2.2 for a prismatic loop ahead of a crack and estimated approximately to range from about 1.2 to 1.9 for representative shear loops inclined to the crack plane. The self energy exceeds the half-full-loop value, corresponding to $m = 1$, and it is observed that this effect increases by $\sqrt{m}$ the predicted loads to nucleate a dislocation loop of the assumed shape from a crack tip.

1. INTRODUCTION

WE PRESENT here calculations of the stress field and self energy for a dislocation loop emerging from a crack tip. The problem is of interest mainly for shear dislocations, in estimating when they may be nucleated from the tip by the concentration of an applied stress field there, and arises also in the study of whether a solid may be regarded as intrinsically cleavable (e.g., RICE and THOMSON, 1974; MASON, 1979; OHR, 1985; LIN and THOMSON, 1986; ANDERSON, 1986; ANDERSON and RICE, 1986). However an exact calculation, within continuum elastic dislocation theory, of the stress field and self energy of a 3D loop at a crack tip has not previously appeared.

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NEARLY CIRCULAR SHEAR MODE CRACKS

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Abstract—In this paper we study the elasticity problems of a planar crack, lying in an infinite three-dimensional solid, the front of which differs only slightly from a circle. The crack system is subjected to loadings that induce shear mode stress intensity factors at the crack front. Quantities such as relative crack surface displacement, in-plane shear mode intensity factor K_2 and anti-plane shear mode intensity factor K_3 are derived in detail. The method used is based on a perturbation technique developed by Rice (*J. Appl. Mech.* **52**, 571–579 (1985)) of calculating the first-order variation of the elastic field of a crack when its front is perturbed from some regular reference geometry. The configurational stability problems of harmonic wave form perturbations of the front of a circular crack under axisymmetric shear loading are studied using the derived formulae. The shape that a planar crack under remote uniform shear loading would take so that the energy release rate distributes uniformly along the crack front is discussed by calculating proper perturbations on a circular crack that meets the above requirement.

INTRODUCTION

For a circular planar crack in an infinite three-dimensional solid, solutions for the stress intensity factors induced along the crack front by various load systems exist in the literature (Tada *et al.*, 1973; Kassir and Sih, 1975; Bueckner, 1977, 1987). Specifically, the solutions for the intensity factor distribution along the circular crack front induced by point force pairs at an arbitrary location on the crack faces, which corresponds to the three-dimensional crack face weight functions of Bueckner (1972) and Rice (1972), generalizing Bueckner's (1970) two-dimensional concepts, are of interest. These solutions were completely derived by Bueckner (1987) for arbitrary point force pairs acting on the crack faces that induce general mixed mode stressing along the crack front, although the solution for a "wedging" force pair that induces mode 1 tension along the crack front was presented earlier by several authors (Tada *et al.*, 1973; Cherepanov, 1979; Bueckner, 1977). Hence by integration of the crack face weight functions we are able to calculate the intensity factors under any load systems for a perfectly circular crack. These solutions, in the limit when the radius of the circular crack approaches infinity, should reduce to the corresponding formulae for a half-plane crack. Solutions for a half-plane crack have been derived by many authors and can be found in Tada *et al.* (1973).

Rice (1985a) developed a method of using the crack face weight function solutions to solve for the elastic field of a crack with a front close to some reference geometry, to first-order accuracy in the deviation of the actual crack from that reference shape. Using that method one can carry out the calculations of the variation of various quantities such as relative crack surface displacements and stress intensity factor distributions when the crack front is perturbed from the reference front to the actual front, if the crack face weight functions for a crack of the reference shape are known in advance.

Rice (1985a) also studied a half-plane tensile crack with a near straight front. In that paper he derived in detail the formulae for the variation in crack opening displacement and stress intensity factors to first-order accuracy in the deviation of the actual crack front from a reference straight line. The shear mode intensity factors for a half-plane crack with a slightly curved crack front were derived by Gao and Rice (1986) using the perturbation method. Gao and Rice (1987) further studied the elasticity problems of somewhat circular tensile cracks. In that paper a full solution, accurate to first order in the deviation of the actual crack front from a circle, is derived for the stress intensity factor distributions and the crack opening displacement. One could verify that those perturbation formulae, in the limit when the radius of the reference crack approaches infinity, also reduce to the corresponding results given by Rice (1985a) for a half-plane crack. Comparison between

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Linear Perturbation Analysis of a Shear-Loaded Asperity

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The elastic problem of a shear-loaded asperity on an infinite fault plane is studied based on a linear perturbation approach developed by Rice (1985). The fault plane is broken everywhere except at an asperity whose shape differs modestly from a circle. The stress concentration along the bounding edge of the asperity due to remotely applied displacements and/or forces is analyzed in terms of in-plane shear mode stress intensity factor K_2 and antiplane shear intensity factor K_3. The asperity configuration corresponds to an external crack which is of importance in understanding the general shear mode fracture of elastic connections. Solutions by Fourier series are developed for asperities subjected to remotely applied displacements and forces. Assuming the fracture process on the fault plane is governed by the intensity of maximum shear stress concentration, we use the perturbation results to study the configurational stability of a circular asperity. It is found that the circular shape is configurationally stable when rigid rotations are fully suppressed at remote field, and the crack front is thus expected to remain circular during quasi-static growth.

INTRODUCTION

In studying the fracture processes at a tectonic fault that leads to an earthquake, the fault is often considered to consist of separated asperities that hold two rough surfaces together. The concept of visualizing an earthquake as the shear rupture of these asperities has been termed the "asperity model" in the literature [e.g., *Madariaga*, 1979; *Lay et al.*, 1982; *Das and Kostrov*, 1983].

The capability of a material to resist fracture is usually assumed a material property, referred to as the fracture toughness, which characterizes the bonding strength along the prospective fracture plane. In general, the toughness value varies at different regions along a fault. Such a heterogeneous nature of fracture resistance results in irregular, complicated shapes of asperities, which poses difficulties in the theoretical study. The lack of an efficient tool in analyzing this geometrical complication partially justifies the development of a simple linear perturbation analysis to calculate the elastic field of an asperity whose shape does not differ much from some regular geometries such as a circle.

Rice [1985] formulated a perturbation approach of solving for the elastic field of a crack with a front close to some reference geometry, to the first-order accuracy in the deviation of the actual crack from that reference shape. The perturbation approach has been applied to study slightly curved half plane cracks and nearly circular internal cracks [*Gao and Rice*, 1986, 1987*a*; *Gao*, 1988]. The tensile-loaded nearly circular connections were studied by *Gao and Rice* [1987*b*]. Here the perturbation analysis is further extended to shear-loaded asperities, which are of interest to earthquake faulting studies.

The asperity configuration corresponds to an external crack with a nearly circular crack front (Figure 1). If a circular asperity is taken as the reference, we may study the actual nearly circular asperity using the perturbation method. For convenience, two coordinate systems, Cartesian x, y, z and cylindrical r, y, θ, are adopted to describe the crack geometry. The y axis is perpendicular to the crack plane, or the fault plane. The origin of the coordinates is located at the center of the reference circular asperity. In the fault plane $y = 0$, one has Cartesian coordinates x, z and polar coordinates r, θ with θ being zero along the positive x axis and increasing to 90° along the positive z axis.

The shear mode stress intensity factors K_α ($\alpha = 2, 3$ corresponds to in-plane shear mode 2 and antiplane shear mode 3, respectively) are used to describe the stress concentration at the crack edges. By definition, $K_\alpha/(2\pi\varepsilon)^{1/2}$ is the asymptotic form of the relevant singular shear stress component for mode α at small distance ε ahead of the crack tip. The following asymptotic formulae relates the stress intensity factors and the relative crack surface displacements (see, e.g., *Rice* [1968])

$$\Delta u_n \sim \frac{8(1-\nu^2)}{E}\left(\frac{\rho}{2\pi}\right)^{1/2} K_2$$
$$\Delta u_t \sim \frac{8(1+\nu)}{E}\left(\frac{\rho}{2\pi}\right)^{1/2} K_3 \qquad (1)$$

where n and t are the normal and tangential directions along the actual crack front with n lying in the crack plane and ρ is the distance as measured from the crack front in the negative normal direction. Equation (1) is understood to be a general asymptotic relation in the vicinity of a crack front. Thus one can extract the stress intensity factors from the near-tip behavior of either the crack tip singular stresses (by definition) or the relative crack surface displacements (by equation (1)).

The key quantities in Rice's perturbation formulation are the "weight functions" associated with the reference crack. The weight function concepts are due to *Bueckner* [1970, 1973] and *Rice* [1972] and have been widely used for elastic crack analysis. For example, the weight functions $h_{\alpha j}(\theta'; \mathbf{r}; a)$ for a circular asperity with radius a are defined as the mode α stress intensity factor induced at the crack front position θ' by a unit point force in j ($j = x, y, z$ or r, y, θ) direction at spatial position $\mathbf{r}$. The closed form solutions for $h_{\alpha j}$ for a circular asperity have been fully derived by *Gao* [1989].

The work presented here is restricted to the static analyses of a single shear-loaded asperity on an infinite fault plane. The stress intensity factors at the bounding edge of the asperity are calculated to the first-order accuracy in the

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WEIGHT FUNCTIONS FOR EXTERNAL CIRCULAR CRACKS

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Abstract—In this paper the three-dimensional (3D) weight functions are derived for external circular cracks. The solution method used is similar to that developed by Bueckner (*Int. J. Solids Structures* **23**, 57-93 (1987)) for internal circular cracks lying in infinite elastic solids. A Papkovitch-Neuber potential is used to represent the tensile mode weight function field. This potential, derived from some known solutions to a mixed boundary value potential problem by Galin (*Contact Problems in the Theory of Elasticity*. School of Physical Sciences and Applied Mathematics, North Carolina State College (1953)), also uniquely determines the shear mode weight function fields. The results for internal circular cracks by Bueckner are presented for comparison and for completeness. For external circular cracks, different forms of the weight functions exist corresponding to different displacement boundary conditions at infinity. The Neuber fields, denoting the elastic fields of an external circular crack due to remote forces and/or moments, are used to determine the weight functions under various boundary conditions. The crack face weight functions, defined as the intensity factors induced by a pair of equal, oppositely sensed unit point forces acting on the upper and lower crack faces, are presented in closed formulae. In the Appendices the present results are checked against some existing solutions, e.g. intensity factor solutions due to the point forces acting along the central axis normal to the crack plane.

INTRODUCTION

The concept of "weight functions" was first introduced by Bueckner (1970) for two-dimensional (2D) elastic crack analysis. In Bueckner's work, the weight functions constitute the displacement field of a special elastic field which he referred to as a "fundamental field". A fundamental field satisfies the Navier displacement equations, equilibrates zero body forces and surface tractions. The displacements of that field are of inverse square root singularity in distance from a crack tip, in contrast to the normal square root dependence of regular displacement fields. Applying Betti's theorem of reciprocity to the fundamental field and the regular elastic field of a crack, Bueckner showed that the weighted average of applied forces with the weight functions gives the crack tip stress intensity factors. This gives a primitive interpretation of the weight functions as the point force solutions for the stress intensity factors. Shortly after Bueckner's work, Rice (1972) developed his weight function concepts in a different way, showing that 2D weight functions could be determined by differentiating the elastic displacement field with respect to crack length; hence the knowledge of a 2D elastic crack solution for any one loading allows the crack solution to be determined for the same body under any other loading systems. Following these works there has been a vast literature on application of weight functions on 2D crack analysis.

The three-dimensional (3D) theory of weight functions, extending Bueckner's 2D concepts, was developed independently by Rice (1972), based on displacement field variations associated to first order with an arbitrary variation in position of the crack front, and by Bueckner (1973), based on a 3D analog of fundamental fields that equilibrate null forces with arbitrary distributions of strength of a normally inadmissible singularity along the crack front. The 3D weight functions not only give stress intensity factors along a crack front for arbitrary body force and surface force distributions, but also determine the first-order variation in the displacement field associated with an arbitrary change in crack front position (Rice, 1985a). The latter property further allows the complete elastic field of a cracked body to be determined by integration over a crack size variable from an uncracked state just before the introduction of the crack, to the actual cracked state.

Rice (1985a) further developed a linear perturbation approach that determines the first-order variation of the elastic field for a crack being slightly perturbed from some simple

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APPLICATION OF 3-D WEIGHT FUNCTIONS—I. FORMULATIONS OF CRACK INTERACTIONS WITH TRANSFORMATION STRAINS AND DISLOCATIONS

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ABSTRACT

IN THIS PAPER we formulate the three-dimensional elastic problem of a half plane crack interacting with zones of transformation strain and dislocations. The stress intensity factors induced at the crack tip are discussed. The analysis is based on RICE's (*Int. J. Solids Struct.* **21**, 781, 1985b) development, using three-dimensional "weight function" theory, of elastic crack tip interactions with internal stress sources and on BUECKNER's (*Int. J. Solids Struct.* **23**, 57, 1987) solution for the complete set of weight functions for a half plane crack. The formulae for the shear mode weight functions by Bueckner are simplified significantly. Explicit formulae are given for the intensity factors induced by arbitrarily distributed 3-D transformation strains, which reduce to the solutions known in the literature for 2-D transformation strains. Results are also specified to calculate the intensity factor distribution due to rectangular and semicircular crack-tip dislocation loops, and compared to those previously estimated by ANDERSON and RICE (*J. Mech. Phys. Solids* **35**, 743, 1987). The three-dimensional results are novel and in part II of this series (GAO and RICE, *J. Mech. Phys. Solids* **37**, 155, 1989) we use the formulations presented here to calculate the self energy of a 3-D shear dislocation loop emerging from the crack tip.

INTRODUCTION

THE ELASTIC interaction between a crack tip and sources of internal stress such as transformation strains and dislocations is of interest in understanding the phenomenon of transformation toughening of materials, such as the toughening of ceramics due to martensitic type transformations of second phase particles (e.g. zirconia, ZrO_2) triggered by the elevated stress field at a crack tip, and also in estimating when dislocation loops may be nucleated from the tip by the stress concentration there. The latter problem arises also in the study of whether a solid may be regarded as intrinsically cleavable (e.g. RICE and THOMSON, 1974; MASON, 1979; OHR, 1985; LIN and THOMSON, 1986; ANDERSON, 1986; ANDERSON and RICE, 1986, 1987). Transformation toughening phenomena have been studied by various ceramists (e.g. PORTER and HEUER, 1977; EVANS and HEUER, 1980) and modeled as the problem of a 2-D crack system subjected to a zone of dilatant phase transformations (e.g. MCMEEKING and EVANS, 1982; BUDIANSKY *et al.*, 1983; ROSE, 1986), the shear, shape and orientation effects of the second phase particles being further accounted for in LAMBROPOULOS (1986). The basic approach in quantitative treatments of this phenomenon is to calculate the reduction of crack tip stress intensity factor by the

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APPLICATION OF 3-D WEIGHT FUNCTIONS—II. THE STRESS FIELD AND ENERGY OF A SHEAR DISLOCATION LOOP AT A CRACK TIP

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(*Received* 26 *February* 1988)

ABSTRACT

THE GENERAL weight function expressions given in GAO (*J. Mech. Phys. Solids* 37, 133, 1989), referred to here as part I, for combined-mode crack-dislocation interaction problems in the three-dimensional regime are applied to solve for the stress field and energy of a shear dislocation loop emerging from the tip of a half-plane crack. The results are compared to the previously proposed approximate estimates for shear loops by ANDERSON and RICE (*J. Mech. Phys. Solids* 35, 743, 1987), who solved exactly for prismatic opening dislocation loops that are co-planar with the crack and also for the analogous 2-D cases of general crack tip-parallel line dislocations. The energy results are presented in terms of a correction factor m, following Anderson and Rice, to the usual estimate of energy for an emergent crack tip loop as half the energy of a full loop (identified as the emergent loop and its image relative to the crack front) in an uncracked solid. For a full circular shear loop the energy is $U = [(2-\nu)\mu b^2 r/4(1-\nu)] \ln (8r/e^2 r_0)$, where r_0 denotes the core cut-off parameter and μ, ν are the shear modulus and Poisson ratio. Thus for a semicircular loop emerging from the crack tip, the energy is expressed as $U = [(2-\nu)\mu b^2 r/8(1-\nu)] \ln (8mr/e^2 r_0)$, where the constant m depends on the orientation angle ψ of the Burgers vector relative to a line normal to the crack tip and the inclination angle ϕ of the dislocated plane relative to the crack plane. The m factors are calculated at selected angles ϕ for rectangular and semicircular loops. This involves multiple numerical integrations based on the weight functions of part I, first to obtain the stress field and then to integrate it over the dislocated area to get the energy, and requires a large amount of computing CPU time. An approximate formula for m is proposed for general inclined dislocation loops, based on known 2-D results for m factors for arbitrary angles ϕ calculated by ANDERSON and RICE (1987) and the 3-D $m(\phi = 0)$ results given here for shear dislocation loops in the crack plane. It compares well to the exact results.

INTRODUCTION

IN PART I (GAO, 1989) we have presented some explicit formulae for calculation of stress intensity factors induced by interaction of transformation strains and dislocations with crack tips. The calculation is based on the three-dimensional weight function solutions by BUECKNER (1987) while the formulation of the problem is based on the analysis of a crack tip interacting with sources of internal stress of RICE (1985) and ANDERSON and RICE (1987).

It remains an interesting topic to study the ductile vs brittle response to cracks in various materials, and this partly includes considerations of whether a solid is intrinsically cleavable. RICE and THOMSON (1974) have proposed that such intrinsic cleavability is determined by the competition between cleavage decohesion and crack

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Shear Stress Intensity Factors for a Planar Crack With Slightly Curved Front

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Recent work (Rice, 1985a) has presented the calculations of the first order variation in an elastic displacement field associated with arbitrary incremental planar advance of the location of the front of a half-plane crack in a loaded elastic full space. That work also indicated the relation of such calculations to a three-dimensional weight function theory for crack analysis and derived an expression for the distribution of the tensile mode stress intensity factor along a slightly curved crack front, to first order accuracy in the deviation of the crack front location from a reference straight line. Here we extend the results on stress intensity factors to the shear modes, solving to similar first order accuracy for the in-plane (Mode 2) and antiplane (Mode 3) shear stress intensity factors along a slightly curved crack front. Implications of results for the configurational stability of a straight crack front are discussed. It is also shown that the concept of line tension, while qualitatively useful in characterizing the crack extension force (energy release rate) distribution exerted on a tough heterogeneity along a fracture path as the crack front begins to curve around it, does not agree with the exact first order effect that is derived here.

Introduction

For a half-plane crack lying in an infinite space, the stress intensity factors due to point force pairs acting on the crack surface have been derived by many authors (Uflyand, 1965; Sih and Liebowitz, 1968; Kassir and Sih, 1973; Bueckner, 1977; Meade and Keer, 1984a; etc) in the case when the crack front lies along a straight line. Hence, by integration, the solution due to arbitrary loading on the crack surface can be found.

Rice (1985a) showed how the knowledge of such solutions enables one to calculate the changes in crack surface displacement distribution, exact to the first order in the deviation of the crack front position from a reference straight line, when the crack front position is altered slightly to lie along a general curved arc in the same plane as that of the crack. He gave full details for the case of tensile (Mode 1) loading and derived an expression for the stress intensity factor K_1 along such a nonstraight crack front (again, exact to the first order). The latter work was motivated by the interesting approach to the wavy crack front problem based on asymptotic expansions by Meade and Keer (1984b), although it turned out that their results required correction.

Here we carry through details of the slightly curved crack front analysis for general shear loading, deriving the analogous expressions for the inplane (Mode 2) and antiplane (Mode 3) stress intensity factors, K_2 and K_3, along a nonstraight crack front.

Crack Surface Displacement

We now present the basic equation for crack surface displacements associated with incremental crack growth, following Rice (1985a).

An infinite, homogeneous, isotropic elastic solid contains a half-plane crack with a straight crack front and is subjected to an "original" load system, consisting of some fixed forces and/or imposed boundary displacements, that induces mixed modes with distributions $K_\alpha^0(z')$ of stress intensity factors along the crack front. Here $\alpha = 1, 2, 3$ and z' denotes the location along the crack front. A Cartesian x, y, z coordinate system is attached such that the crack plane lies on $y = 0$ and the z axis lies along the crack front (Fig. 1).

Now imagine that the original loading is supplemented by a set of concentrated force pairs $\pm P_j$, $j = x, y, z$, acting at $x, 0^+, z$ and $x, 0^-, z$ resulting in opening, inplane shear and antiplane shear relative displacements of the crack surface. Let $\Delta u_j(x, z)$ be the relative displacements of crack surfaces at the load location. (These are unbounded for point forces; see Rice (1985a) for a refinement of the argument by distributing the forces over finite discs whose radius is later allowed to approach zero.) Suppose that under the combined load system described, the crack front is advanced normal to itself by some infinitesimal variable distance $\delta a(z')$, where z' is the location

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Nearly Circular Connections of Elastic Half Spaces

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In this paper we solve the elasticity problem of two elastic half spaces that are joined together over a region that does not differ much from a circle, i.e., the problem of an external planar crack leaving a nearly circular uncracked connection. The method we use is based on the perturbation technique developed by Rice (1985) for solving the elastic field of a crack whose front deviates slightly from some reference geometry. Quantities such as crack opening displacement and stress intensity factor are derived in detail to the first order of accuracy in the deviation of the shape of the connection from a circle. In addition, some results such as the crack face weight functions and Green's functions for a perfectly circular connection are also discussed under various boundary conditions at infinity. The formulae derived are used to study the configurational stability problem for quasistatic growth of an external circular crack. The results, derived when the crack front is perturbed from circular in a harmonic wave form and is subjected to axisymmetric loading, suggest that a perturbation of wavenumber higher than one is configurationally stable under all boundary conditions at infinity. The perturbation with wavenumber equal to one, which corresponds to a translational shift of the geometric center of the circular connection, turns out to be configurationally stable if any rotation in the remote field is suppressed and configurationally unstable if there is no such restraint.

Introduction

Rice (1985) developed a method of solving the elasticity problem of a planar crack whose front differs slightly in location from that of some reference geometry. It has been applied to cases such as semi-infinite planar cracks with slightly nonstraight fronts (Rice, 1985; Gao and Rice, 1986) and internal somewhat circular cracks (Gao and Rice, 1987). The latter work (Gao and Rice, 1987) has shown that the perturbation method is not only convenient but also remarkably accurate in determining crack opening displacement and stress intensity factors for crack configurations that differ moderately from a circular reference geometry. The internal circular crack problem was addressed much earlier in a perturbation sense by Panasyuk (1962), and Gao and Rice (1987) compare their approach to his. Rice's perturbation method can be carried out immediately for a tensile crack if the solution for the stress intensity factor distribution is known along the reference crack front due to a pair of concentrated wedging forces acting to open the crack at an arbitrary location on its surfaces. Such a point force solution, sometimes called the crack face weight function after Bueckner (1970, 1973) and Rice (1972), was derived by Stallybrass (1981) for an external circular crack, i.e., a circular connection between elastic half-spaces under a traction free boundary condition at infinity. Following Stallybrass's work we are also able to clarify ambiguities in some previously proposed solutions in the literature (e.g., Kassir and Sih, 1975; Tada et al., 1973).

In this paper we therefore solve for the crack opening displacement and tensile mode stress intensity factor for a slightly noncircular connection. The notation δ (F) is used in what follows to denote the variation in some field variable F from its form for the reference circular crack to that for the perturbed crack shape.

Consider two isotropic, homogeneous three-dimensional elastic semi-infinite solids joined over some slightly noncircular connection of bounding contour c. A Cartesian coordinate system x, y, z is attached so that the joining planes lie on $y = 0$ and the origin of the coordinate system is assumed to coincide with the center of some convenient reference circle. This configuration forms an external crack with its front c described by some function $a(s)$ where $a(s)$ is the distance from the origin of the coordinate system to the position s along the crack front; $a(s)$ is nearly constant, and is constant on the reference circle. The crack system is subjected to some distribution of fixed forces that induce "Mode 1" tension along the crack front. We may note that in this case when the crack grows into the connecting ligament, $a(s)$ decreases. Therefore, we represent the crack growth from the reference circular shape to the actual shape by $-\delta a(s)$. In this circumstance it can be shown, following Rice (1985), that the variation in opening displacement $\Delta u(x,z)$ between upper and lower crack surfaces at location x, z, when the crack front is

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Somewhat circular tensile cracks

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Abstract

In this paper we apply the method developed by Rice [1], of solving for the elastic field of a crack with a front perturbed from some reference shape, to solve the elasticity problems of somewhat circular planar tensile cracks under arbitrary load distributions. The method is based on a known solution for the stress intensity factor along a circular crack due to a pair of wedge-opening point forces on its surfaces. A full solution, accurate to first order in the deviation from a circular shape, is derived for the stress intensity factor and the crack opening displacement distributions. The results of a perturbation in a harmonic wave form suggest that a circular crack, under axially symmetric loading, can be configurationally unstable (not grow as a circle) for loadings that increase in intensity with distance from the center. Circular cracks with harmonic shape perturbations are found to have the same form of variation of the stress intensity factor with arc length along the crack edge (to first order accuracy) as found in previous work for a half plane crack. As a test case for the perturbation solution, an elliptical planar tensile crack under uniform tension is viewed as being perturbed from a circular crack. Results derived from the perturbation formulae through numerical evaluation are compared with the exact solutions existing in the literature. The perturbation results show a very good match with the exact solutions even when the semi-axis lengths of the elliptical crack differ by a factor of two (and by as much as a factor of three when special choices of the reference circular crack location are made). This suggests that the perturbation procedure presented here, while theoretically exact only to first order, can be used to produce acceptable results for some planar cracks whose shapes deviate appreciably from a circle.

1. Introduction

We study in this paper planar tensile cracks whose tips lie along arcs that are slightly distorted from circles. Of particular interest is how the distributions of the stress intensity factor and the crack face opening displacement change to the first order of accuracy when the front of an arbitrarily loaded crack is perturbed from a circular shape. A full solution for these quantities is given in detail.

The method that we use, developed by Rice [1] to solve for the elastic field of a crack with a front perturbed from some reference geometry, can be carried out at once if a certain solution is known for a crack of that reference geometry. For tensile cracks this is the solution for the stress intensity factor distribution along the reference crack front due to a pair of wedging forces acting to open it at an arbitrary location on its surfaces. Fortunately, the requisite point force solution can be developed from the work of Galin [2] for an internal circular crack in an infinite body. It is given by Tada et al. [3] and Cherepanov [4].

Rice [1] applied his method to the half-plane crack in an infinite body. Using the point force solution for that case he calculated the crack surface displacement distribution, exact to first order in the perturbation of the crack front location from a reference straight line (he also gave a direct ab initio first order solution for the elastic field of the perturbed crack). He further derived an expression for the stress intensity factor K along such a non-straight crack front (again exact to first order) and this led to a discussion of the wavy crack front problem, motivated by the study of the same problem by Meade and Keer [5].

Our work here is based on the following result. Consider that an infinite three dimensional elastic solid contains a planar crack with smooth bounding contour c along the crack front. We

A Rate-Independent Constitutive Theory for Finite Inelastic Deformation[4]

R. Hill[5] and J. R. Rice.[6] Hill (1968) showed that the Il'yushin (1961) inequality, when interpreted at finite strain as the postulate that nonnegative stress working is done in any strain cycle, leads to "normality" in work-conjugate stress and strain variables for rate-insensitive elastic-plastic solids. That is, the plastic part of a general strain increment (which can be interpreted as the residual strain increment after an imagined elastic unloading to the same stress as at the start of that increment) then has direction normal to the yield surface in stress space when that surface is smooth, and direction within the cone of limiting normals at a vertex. Similarly, an analogously defined plastic part of a stress increment is normal to the yield surface in strain space. The paper under discussion by Carroll (1987) notes the discovery of such a result by Naghdi and Trapp (1975) based on their "work assumption," which is apparently the same as the Il'yushin (1961) postulate.

[4] By M. M. Carroll and published in the March, 1987, issue of ASME Journal of Applied Mechanics, Vol. 54, No. 1, pp. 15-21.

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Neither Carroll nor the various papers by Naghdi and coworkers that he cites mention the paper by Hill (1968). Carroll does, however, discuss a paper by Hill and Rice (1973). He states: "Hill and Rice (1973) discussed some implications of Il'yushin's postulate . . . and they also discussed normality conditions . . . Their discussion has apparently created a mistaken impression that they have derived the normality conditions as a consequence of the Il'yushin postulate . . . , although this claim is not made in the paper by Hill and Rice (1973)." Then in the following discussion he asserts: "The analysis of Hill and Rice, which treats only the first order contribution to the external work in the cycle of deformation . . . does not provide a proof of the normality condition."

It is difficult to understand these contentions. Hill and Rice (abbreviated H-R) showed in subsection 3.2 of their paper that the Il'yushin inequality, when applied to strain cycles with infinitesimal plastic strain accumulation (corresonding, e.g., to change $d\mathbf{K}$ in Carroll's tensor parameter $\mathbf{K}$ characterizing plastic state) implies the inequality

$$[d^p\Psi]_{\mathbf{S}_*}^{\mathbf{S}_\circ} = d^p\Psi(\mathbf{S}_\circ,\mathbf{K}_\circ,d\mathbf{K}) - d^p\Psi(\mathbf{S}_*,\mathbf{K}_\circ,d\mathbf{K}) < 0 \tag{1}$$

with $d^p\Psi(\mathbf{S},\mathbf{K}_\circ,d\mathbf{K}) = \Psi(\mathbf{S},\mathbf{K}_\circ + d\mathbf{K}) - \Psi(\mathbf{S},\mathbf{K}_\circ)$. This is one of the H-R inequalities (27), changed to Carroll's notation and special form of representing the plastic state; Ψ is the complementary energy, a function of stress $\mathbf{S}$ and $\mathbf{K}$ (which changes only during plastic response) such that strain $\mathbf{E} = \partial\Psi(\mathbf{S},\mathbf{K})/\partial\mathbf{S}$; the Il'yushin strain cycle begins at state $\mathbf{E}_\circ,\mathbf{S}_\circ,\mathbf{K}_\circ$ within the elastic domain and plastic deformation associated with $d\mathbf{K}$ commences at state $\mathbf{E}_*,\mathbf{S}_*,\mathbf{K}_*$ on the yield surface (and $\mathbf{K}_* = \mathbf{K}_\circ$).

H-R identified inequality (1) above as a "global" form of one of their inequalities (24), where the H-R inequalities (24) are local in a sense explained shortly. The latter inequalities had been shown in the immediately preceding subsection of their paper to be inequalities that ensure normality. It was taken as evident that since the Il'yushin work assumption implied certain global inequalities, it also implied the local forms of the same inequalities, and since the local forms implied normality, this proved that the Il'yushin inequality implied normality.

If H-R did not see the necessity to state any more explicitly that they had ". . . derived the normality conditions as a consequence of the Il'yushin postulate," such would presumably be because they had stated explicitly at the outset of their subsection 3.2 that Hill (1968) had already established that connection, and thus they would reasonably think a concisely outlined derivation to be sufficient.

The local inequality (24) of H-R just discussed, related to (1) above, reads

$$\delta(d^p\Psi) = d^p\Psi(\mathbf{S}_* + \delta\mathbf{S},\mathbf{K}_\circ,d\mathbf{K}) - d^p\Psi(\mathbf{S}_*,\mathbf{K}_\circ,d\mathbf{K}) < 0 \tag{2}$$

where $\delta\mathbf{S}$ denotes any infinitesimal stress change directed into the elastic domain from the point $\mathbf{S}_*$ on the yield surface in stress space from which the plastic deformation associated with $d\mathbf{K}$ commences. The inequality is local in the sense that (2) is the form of (1) when we identify $\delta\mathbf{S}$ as $\mathbf{S}_\circ - \mathbf{S}_*$ and write (1) for stresses $\mathbf{S}_\circ$ within the elastic domain arbitrarily close to $\mathbf{S}_*$. Inequality (2) implies normality because the plastic part

$$d^p\mathbf{E} = \mathbf{E}(\mathbf{S}_*,\mathbf{K}_\circ + d\mathbf{K}) - \mathbf{E}(\mathbf{S}_*,\mathbf{K}_\circ)$$

of the overall strain increment during which $d\mathbf{K}$ accumulates satisfies $d^p\mathbf{E} = \partial[d^p\Psi(\mathbf{S},\mathbf{K}_\circ,d\mathbf{K})]/\partial\mathbf{S}$ at $\mathbf{S} = \mathbf{S}_*$, so that inequality (2) is equivalently written as

$$\delta\mathbf{S}\cdot d^p\mathbf{E} < 0. \tag{3}$$

The latter inequality, corresponding to H-R inequality (22), implies normality of $d^p\mathbf{E}$ to the yield surface in $\mathbf{S}$ space.

H-R also addressed the conjugate formulation in terms of a yield surface in strain space. In terms of $d^p\mathbf{S} = \mathbf{S}(\mathbf{E}_*, \mathbf{K}_\circ + d\mathbf{K}) - \mathbf{S}(\mathbf{E}_*,\mathbf{K}_\circ)$, they showed that the conjugate inequality $\delta\mathbf{E}\cdot d^p\mathbf{S} > 0$ followed from (3) where $\delta\mathbf{E}$ points into the elastic domain from $\mathbf{E}_*$. This implies normality of $d^p\mathbf{S}$ to the yield surface in $\mathbf{E}$ space.

Inequalities (2) and (3) are bilinear forms in $\delta\mathbf{S}$ and $d\mathbf{K}$; the two sets of infinitesimals consisting of $\delta\mathbf{S},\delta\mathbf{E}$ (strain related elastically to $\delta\mathbf{S}$) and of $d\mathbf{K}$, $d^p\mathbf{E}$, $d^p\mathbf{S}$ are independent of one another within the constraints of their definitions. Carroll obscures this issue in a recapitulation of H-R which uses the same presymbol Δ for the various infinitesimals, as $\Delta\mathbf{E}$ for $\delta\mathbf{E}$ and $\Delta\mathbf{K}$ for $d\mathbf{K}$. He then tacitly assumes that $\Delta\mathbf{E}(=\delta\mathbf{E})$ and $\Delta^p\mathbf{E}\ (= d^p\mathbf{E})$ are of the same order. In that circumstance he observes that since the result of the Il'yushin work integral coincides with the left side of inequality (1), apart from sign, only to first order in $\Delta\mathbf{K}(= d\mathbf{K})$, that particular contribution to the work integral is therefore of order $\Delta\mathbf{E}\Delta\mathbf{K}$ in the case he considers, and therefore is of the same order as terms of order $(\Delta\mathbf{K})^2$ already neglected in evaluating the work integral. His statements in this regard are correct but not relevant to the H-R presentation.

Rather, in that presentation it is evident that inequality (1) is a valid consequence of the Il'yushin work inequality only to first order in $d\mathbf{K}$, where $d\mathbf{K}$ can be arbitrarily small, the context evidently being that the order of $d\mathbf{K}$ must be negligible compared to that of the difference $\mathbf{S}_\circ - \mathbf{S}_*$ (i.e., $|d^p\mathbf{E}| << |\mathbf{E}_\circ - \mathbf{E}_*|$ to use analogous quantities, where we define $|\mathbf{E}| = (E_{ij}E_{ij})^{1/2}$). Inequality (2), which has equivalent form as inequality (3), is thus a valid consequence to first order in $\delta\mathbf{S} = \mathbf{S}_\circ - \mathbf{S}_*$ for arbitrarily small $\delta\mathbf{S}$ provided that one remembers that, for any given $\delta\mathbf{S}$, the order of $d\mathbf{K}$ is to be taken arbitrarily small by comparison. That is, the order of $d^p\mathbf{E}$ in (3) must always be regarded as being arbitrarily small by comparison that of $\delta\mathbf{S}$; $|d^p\mathbf{E}| << |\delta\mathbf{E}|$, where $\delta\mathbf{E}$ is the strain associated elastically with $\delta\mathbf{S}$. This wide disparity of the order of the terms in (3) is of no consequence since, for inferring normality, one cares only about the directions and not the magnitude of $d^p\mathbf{E}$ and $\delta\mathbf{S}$. Carroll's unhappiness with the H-R presentation is self-generated by his choice of notations $\Delta\mathbf{K}$ for $d\mathbf{K}$ and $\Delta\mathbf{S}$ for $\delta\mathbf{S}$ which is inconsistent with the independence of these quantities and converts the bilinear form (3) to a quadratic form.

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DYNAMIC GROWTH OF ANTI-PLANE SHEAR CRACKS IN IDEALLY PLASTIC CRYSTALS

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A near-tip asymptotic analysis is given for the stress and deformation field near the tip of crack propagating dynamically under anti-plane shear in an ideally plastic single crystal. A particular class of orientations of the crack relative to the crystal is considered so that the yield locus is of simple diamond shape (relative to directions along and perpendicular to the crack) in the plane of the anti-plane shear stresses. The near-tip solution is shown to consist of sectors which carry constant stresses, at yield levels, corresponding to adjacent vertices on the diamond-shaped yield locus, and which are joined along an elastic–plastic shock discontinuity. All plastic flow in the near-tip region occurs in the shock. Plastic strains and particle velocity are finite at the crack tip. The plastic strain is proportional to the elastic strain at onset of yielding and is inversely proportional to the elastic Mach number associated with the speed of crack growth.

1. Introduction

Dynamic crack growth in ideally plastic single crystals is analyzed here for geometries and orientations such that two-dimensional states of anti-plane shear constitute a possible deformation field. The analysis is asymptotic; the limit $r \to 0$ is considered where r is distance from the moving crack tip. Cases of stationary and quasistatically growing anti-plane cracks for different orientations in f.c.c. and b.c.c. crystals were solved by Rice and Nikolic (1985). Here inertia effects are taken into account for the growing crack. The material yields according to the attainment of a critical value for the resolved shear stress on one or more different slip systems in a crystal. Since for a perfectly plastic material the shear wave velocity for an appropriate direction of straining is zero, the crack growth is supersonic even at small speeds. Thus, the inertia terms in the basic equations may have a significant effect on the nature of near tip fields. It may also be expected that the quasistatic solution includes features that will not be present in dynamic results. For Mode III crack growth in *isotropic* ideally plastic solids, this was shown by Slepyan (1976) and confirmed by Achenbach and Dunayevsky (1981), and Freund an Douglas (1982). These authors discovered that the Mode III dynamic solution, unlike the quasistatic one for a growing crack, predicts no elastic unloading sectors and the entire field around the crack tip is plastic. The shear strain has a logarithmic singularity. The solutions considered here are different because we consider the behavior of single crystals, not the isotropic material.

For the stationary crack cases in crystals the platic zone at a crack tip collapses into discrete planes of displacement and stress discontinuity emanating from the tip. For the quasistatically growing crack these same planes also constitute collapsed plastic zones in which velocity and plastic strain discontinuities occur but across which the stresses and anti-plane displacement are fully continuous. For the dynamic growth case considered in the present work the configuration of the stress field around the crack tip is expected to be quite different. We considered different types of near-tip solutions to the equations governing dynamic growth of a crack in anti-plane shear, like elastic and plastic sectors, both of constant and variable stresses. We conclude that the whole near-tip field around the crack tip is plastic (or is at least stressed to a level meeting the yield condition). For the range of the coordinate angle θ of 0 to 180° the solution consists of two plastic sectors of constant stresses with the boundary between

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TENSILE CRACK TIP FIELDS IN ELASTIC-IDEALLY PLASTIC CRYSTALS

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Crack tip stress and deformation fields are analyzed for tensile-loaded ideally plastic crystals. The specific cases of (0 1 0) cracks growing in the [1 0 1] direction, and (1 0 1) cracks in the [0, 1, 0] direction, are considered for both fcc and bcc crystals which flow according to the critical resolved shear stress criterion. Stationary and quasistatically growing crack fields are considered. The analysis is asymptotic in character; complete elastic-plastic solutions have not been determined. The near-tip stress state is shown to be locally constant within angular sectors that are stressed to yield levels at a stationary crack tip, and to change discontinuously from sector to sector. Near tip deformations are not uniquely determined but fields involving shear displacement discontinuities at sector boundaries are required by the derived stress state. For the growing crack both stress and displacement must be fully continuous near the tip. An asymptotic solution is given that involves angular sectors at the tip that elastically unload from, and then reload to, a plastic state. The associated near-tip velocity field then has discontinuities of slip type at borders of the elastic sectors. The rays, emanating from the crack tip, on which discontinuities occur in the two types of solutions are found to lie either parallel or perpendicular to the family of slip plane traces that are stressed to yield levels by the local stresses. In the latter case the mode of concentrated shear along a ray of discontinuity is of kink type. Some consequences of this are discussed in terms of the dislocation generation and motion necessary to allow the flow predicted macroscopically.

Introduction

An asymptotic analysis of the crack tip stress and deformation field is presented for plane-strain tensile cracks in elastic-ideally plastic single crystals. Such crystals are assumed to have a limited set of possible slip systems and to have a critical resolved shear stress for plastic flow to occur on each. Here attention is limited to two specific crack orientations in face centered and body centered cubic crystals, although the analysis techniques are applicable to other orientations too.

One orientation considered is such that the crack plane is (0 1 0), i.e., parallel to a face of the reference cubic cell, and the crack tip lies along the face diagonal direction $[1\ 0\ \bar{1}]$; the crack grows along the perpendicular face diagonal, [1 0 1]. See Fig. 1 for the fcc case and Fig. 2 for bcc. The analysis in all but the second to last section of the paper is discussed relative to that orientation. It also happens to provide the solution for a second orientation considered, which still has the crack tip along the $[1\ 0\ \bar{1}]$ face diagonal but which has the crack plane as the (1 0 1) plane so that [0 1 0] is the direction of crack growth. Thus the second orientation has the crack line rotated 90° anticlockwise from what is pictured in Figs. 1(b) and 2(b). These two crack orientations are often, but not universally, encountered in experimental studies of cracking, whether by rapid cleavage, fatigue or chemically assisted crack growth, in ductile fcc and bcc metals (Tetelman and Robertson, 1963; Tetelman and Johnston, 1965; Neumann, 1974a, b; Garrett and Knott, 1975; Hecker et al., 1978; Rieux et al., 1979; Neumann et al., 1979; Vehoff and Neumann, 1979, 1980; Lynch, 1983, 1985; Sieradzki et al., 1984; Sieradzki and Newman, 1985; Pugh, 1985; Wang, 1987).

Figures 1 and 2 also show the slip systems which are assumed to be active in relaxing the crack tip stress concentration.

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Two general integrals of singular crack tip deformation fields

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Abstract. The Eshelby tensor E has vanishing divergence in a homogeneous elastic material, whereas the invariance of the crack tip J integral suggests, in accord with known solutions, that the product rE will have a finite limit at the tip. Here r is distance from the tip. These considerations are shown to lead to two general integrals of the equations governing singular crack tip deformation fields. Some of their consequences are discussed for analysis of crack tip fields in linear and nonlinear materials.

Introduction

Consider a homogeneous elastic solid, linear or nonlinear, containing a planar crack on $x_2 = 0, x_1 < 0$ (Figure 1). The solid is loaded such that the near tip field is two-dimensional in the x_1, x_2 plane, thus consisting of some combination of in-plane and anti-plane deformation with stresses $\sigma_{ij} = \sigma_{ij}(x_1, x_2)$ and displacements $u_k = u_k(x_1, x_2)$. Here Latin indices $i, j, k, \ldots$ range over 1, 2, 3, whereas Greek indices $\alpha, \beta, \gamma, \ldots$ range over 1, 2 only. The analysis which follows applies also to elastic-plastic solids treated within the approximation of the "deformation", or "total strain", formulation.

The governing equations are the three equilibrium conditions (in the absence of body forces)

$$\sigma_{\alpha j,\alpha} = 0 \qquad (1)$$

($f_{,\alpha} = \partial f/\partial x_\alpha$) and the stress-displacement gradient relations

$$\sigma_{ij} = \partial W/\partial u_{j,i} \qquad (2)$$

where W is the stress work (or strain energy) density, and is a function of displacement gradients that is properly invariant to rigid rotations. Further, a consequence of these equations is that the integrals

$$J_\alpha \equiv \oint_C n_\beta E_{\beta\alpha}\, \mathrm{d}s = 0 \qquad (3)$$

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Weight Function Theory for Three-Dimensional Elastic Crack Analysis

REFERENCE: Rice, J. R., **"Weight Function Theory for Three-Dimensional Elastic Crack Analysis,"** *Fracture Mechanics: Perspectives and Directions (Twentieth Symposium), ASTM STP 1020,* R. P. Wei and R. P. Gangloff, Eds., American Society for Testing and Materials, Philadelphia, 1989, pp. 29–57.

ABSTRACT: Recent developments in elastic crack analysis are discussed based on extensions and applications of weight function theory in the three-dimensional regime. It is shown that the weight function, which gives the stress intensity factor distribution along the crack front for arbitrary distributions of applied force, has a complementary interpretation: It characterizes the variation in displacement field throughout the body associated, to first order, with a variation in crack-front position. These properties, together with the fact that weight functions have now been determined for certain three-dimensional crack geometries, have allowed some new types of investigation. They include study of the three-dimensional elastic interactions between cracks and nearby or emergent dislocation loops, as are important in some approaches to understanding brittle versus ductile response of crystals, and also the interactions between cracks and inclusions which are of interest for transformation toughening. The new developments further allow determination of stress-intensity factors and crack-face displacements for cracks whose fronts are slightly perturbed from some reference geometry (for example, from a straight or circular shape), and those solutions allow study of crack trapping in growth through a medium of locally nonuniform fracture toughness. Finally, the configurational stability of cracking processes can be addressed: For example, when will an initially circular crack, under axisymmetric loading, remain circular during growth?

KEY WORDS: fracture mechanics, elasticity theory, weight functions, stress intensity factors, dislocation emission, crack-defect interactions, configurational stability, crack trapping

Bueckner introduced the concept of "weight functions" for two-dimensional elastic crack analysis in 1970 [*1*]. His weight functions satisfy the equations of linear elastic displacement fields, but they equilibrate zero body and surface forces and have a stronger singularity at the crack tip than would be admissible for an actual displacement field. The worklike product of an arbitrary set of applied forces with the weight function gives the crack-tip stress intensity factor induced by those forces. Bueckner's contribution led to what is now a vast literature on two-dimensional elastic crack analysis. One of the earliest works of that literature was a 1972 paper by the writer [*2*] which showed that weight functions could be determined by differentiating known elastic displacement field solutions with respect to crack length. It was also shown [*2*] that knowledge of a two-dimensional elastic crack solution, as a function of crack length, for any one loading enables one to determine directly the effect of the crack on the elastic solution for the same body under any other loading system.

The subject here is three-dimensional weight-function theory. Foundations of the three-dimensional theory were given independently by the writer, in the Appendix of Ref *2*, based

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Crack tip fields in ductile crystals

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Abstract. Results on the asymptotic analysis of crack tip fields in elastic-plastic single crystals are presented and some preliminary results of finite element solutions for cracked solids of this type are summarized. In the cases studied, involving plane strain tensile and anti-plane shear cracks in ideally plastic fcc and bcc crystals, analyzed within conventional small displacement gradient assumptions, the asymptotic analyses reveal striking discontinuous fields at the crack tip.

For the stationary crack the stress state is found to be locally uniform in each of a family of angular sectors at the crack tip, but to jump discontinuously at sector boundaries, which are also the surfaces of shear discontinuities in the displacement field. For the quasi-statically growing crack the stress state is fully continuous from one near-tip angular sector to the next, but now some of the sectors involve elastic unloading from, and reloading to, a yielded state, and shear discontinuities of the velocity field develop at sector boundaries. In an anti-plane case studied, inclusion of inertial terms for (dynamically) growing cracks restores a discontinuous stress field at the tip which moves through the material as an elastic-plastic shock wave. For high symmetry crack orientations relative to the crystal, the discontinuity surfaces are sometimes coincident with the active crystal slip planes, but as often lie perpendicular to the family of active slip planes so that the discontinuities correspond to a kinking mode of shear.

The finite element studies so far attempted, simulating the ideally plastic material model in a small displacement gradient type program, appear to be consistent with the asymptotic analyses. Small scale yielding solutions confirm the expected discontinuities, within limits of mesh resolution, of displacement for a stationary crack and of velocity for quasi-static growth. Further, the discontinuities apparently extend well into the near-tip plastic zone. A finite element formulation suitable for arbitrary deformation has been used to solve for the plane strain tension of a Taylor-hardening crystal panel containing a center crack with an initially rounded tip. This shows effects due to lattice rotation, which distinguishes the regular versus kinking shear modes of crack tip relaxation, and holds promise for exploring the mechanics of crack opening at the tip.

1. Introduction

This paper summarizes recent analytical and numerical investigations into the nature of the near-crack-tip stress and deformation fields in ductile single crystals. Ductile crystals deform plastically by the motion of dislocations on a limited set of slip systems. A continuum representation of this plastic deformation consistent with the Schmid rule, which states that flow on a system is activated when the shear stress resolved on that system reaches a critical value, is used in the analyses to be presented. This formulation leads to a yield surface in stress space consisting of planar facets joined at vertices and to an "associated" plastic straining relation.

General methods of constructing asymptotic near-tip fields for such crystals, with either stationary or quasi-statically growing cracks, have been obtained in the ideally plastic case for both anti-plane strain (mode III: [1]) and tensile plane strain (mode I: [2]) cracks. The results, as illustrated for common crack orientations in fcc and bcc crystals, lead to

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CRACK TIP SINGULAR FIELDS IN DUCTILE CRYSTALS WITH TAYLOR POWER-LAW HARDENING. I: ANTI-PLANE SHEAR

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ABSTRACT

ASYMPTOTIC singular solutions of the HRR type are presented for anti-plane shear cracks in ductile crystals. These are assumed to undergo Taylor hardening with a power-law relation between stress and strain at sufficiently large strain. Results are given for several crack orientations in fcc and bcc crystals. The near-tip region divides into angular sectors which are the maps of successive flat segments and vertices on the yield locus. Analysis is simplified by use of new general integrals of crack tip singular fields of the HRR type. It is conjectured that the single crystal HRR fields are dominant only over part of the plastic region immediately adjacent to the crack tip, even at small scale yielding, and that their domain of validity vanishes as the perfectly plastic limit is approached. This follows from the fact that while in the perfectly plastic limit the HRR stress states approach the correct discontinuous distributions of the complete elastic–ideally plastic solutions for crystals (RICE and NIKOLIC, *J. Mech. Phys. Solids* **33**, 595 (1985)), the HRR displacement fields in that limit remain continuous. Instead, the complete elastic–ideally plastic solutions have discontinuous displacements along planar plastic regions emanating from the tip in otherwise elastically stressed material. The approach of the HRR stress fields to their discontinuous limiting distributions is illustrated in graphical plots of results. A case examined here of a fcc crystal with a crack along a slip plane is shown to lead to a discontinuous near-tip stress state even in the hardening regime.

Through another limiting process, the asymptotic solution for the near-tip field for an isotropic material is also derived from the present single crystal framework.

INTRODUCTION

THE PRESENT article analyzes singular near-tip stress and deformation fields for stationary anti-plane shear (mode III) loaded cracks in strain hardening ductile crystals. It is assumed that the crystals deform by shear on a set of allowable slip systems according to the Schmid rule. That is, plastic flow occurs on a given system once the resolved shear stress on that system reaches a critical value. In addition, the critical shear strengths are assumed to obey Taylor hardening (all systems harden equally) with a power-law relation between stress and strain at sufficiently large strain. Thus, the yield surfaces in stress space, being the inner envelope of the planar yield surfaces for individual slip systems, reduce to self-similar polygons in the two-dimensional anti-plane shear stress plane. The yield surface is a fixed polygon in the space of the ratio of the stresses to the critical shear strength.

In the near-tip field, it is anticipated that the elastic strains are relatively small and ignorable. Hence the entire strain vector can be identified with the plastic strains.

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CRACK TIP SINGULAR FIELDS IN DUCTILE CRYSTALS WITH TAYLOR POWER-LAW HARDENING. II: PLANE STRAIN

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ABSTRACT

AN ASYMPTOTIC singular solution of the HRR type is presented for mode I tensile cracks in ductile single crystals. These are assumed to undergo Taylor hardening with a power-law relation between stress and strain at sufficiently large strain. Results are given for a crack on the (010) plane with its tip along the [10$\bar{1}$] direction, and for a crack on the (101) plane with its tip along the same [10$\bar{1}$] direction in a fcc crystal. The yield surfaces for both of these orientations are identical and thus, for the "small strain" formulation, the same macroscopic solution applies to both. The near-tip region is divided into angular sectors which are maps of successive flat segments and vertices of the yield surface. While the solution here involves 14 different sectors referring to stress states corresponding to flat and vertex segments of the yield locus, RICE's (*Mech. Mater.* **6**, 714, 1987) asymptotic solution for the elastic–ideally plastic crystals involved only 7 sectors which mapped into the vertex points of the yield surface. The perfectly plastic limit of the HRR fields here reduce to 7 stress states of RICE (1987). In this limit, the HRR displacement fields remain continuous resulting in a discontinuous yet bounded and nonzero strain field. In contrast, the elastic–ideally plastic solutions have discontinuous shear displacements across sector boundaries. Furthermore the contours of constant effective strain here have various peaks and troughs at sector boundaries and lean backward relative to the direction of crack growth. Conversely, in the recent finite element solutions for elastic–ideally plastic single crystals by Hawk (preliminary summary of results is included in RICE *et al.*, *Int. J. Fracture*, in press, 1989), the plastic zones lean forward and the strain field is consistent with a Dirac singular form similar to RICE's (1987). Thus it is conjectured that, similar to the anti-plane shear case of RICE and SAEEDVAFA (*J. Mech. Phys. Solids* **36**, 189, 1988), the single crystal HRR fields are dominant only over part of the plastic region immediately adjacent to the crack tip, and that their domain of validity vanishes as the perfectly plastic limit is approached.

INTRODUCTION

RICE (1987) presented an asymptotic solution for the stress and deformation field very near the tip of a mode I crack in an ideally plastic ductile crystal. Two specific orientations, a crack on the (010) cube face with its tip along the [10$\bar{1}$] face diagonal and a crack on the (101) plane with its tip along the same diagonal, were considered in fcc and bcc crystals for both stationary and quasi-statically growing cracks. In the case of stationary cracks, the stress field was found to be piecewise constant in angular sectors mapping into vertex points of the yield surface and changed discontinuously between sectors. The displacement and strain fields were not fully determined, although it was shown that there must be a shear displacement discontinuity across the sector boundaries for elastic–plastic crystals. The full solution could only be